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1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e .

(4)



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2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

(7)



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3. Given that

$$2 \cos(x + 50)^\circ = \sin(x + 40)^\circ$$

(a) Show, without using a calculator, that

$$\tan x^\circ = \frac{1}{3} \tan 40^\circ \tag{4}$$

(b) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos(2\theta + 50)^\circ = \sin(2\theta + 40)^\circ$$

giving your answers to 1 decimal place. (4)



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4. $f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$

- (a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$. (5)

- (b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5} e^{-x}$ (1)

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

- (c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5} e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. (3)

- (d) Give an accurate estimate for α to 2 decimal places, and justify your answer. (2)



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5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y . (2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} \quad (4)$$

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x . Give your answer in its simplest form. (4)



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6. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1)$, $-1 < x < 2$ (5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers. (5)



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7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

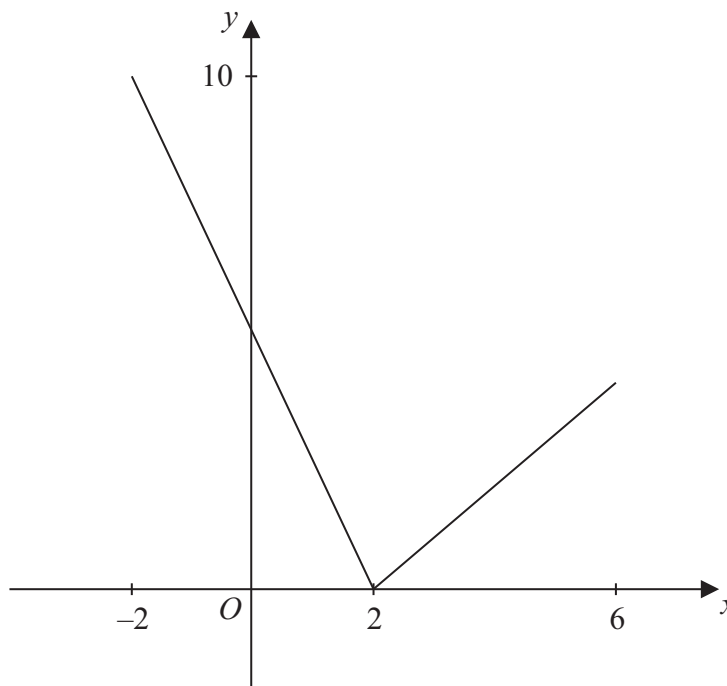


Figure 1

- (a) Write down the range of f . (1)
- (b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$ (3)
- (d) Solve the equation $gf(x) = 16$ (5)



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8.

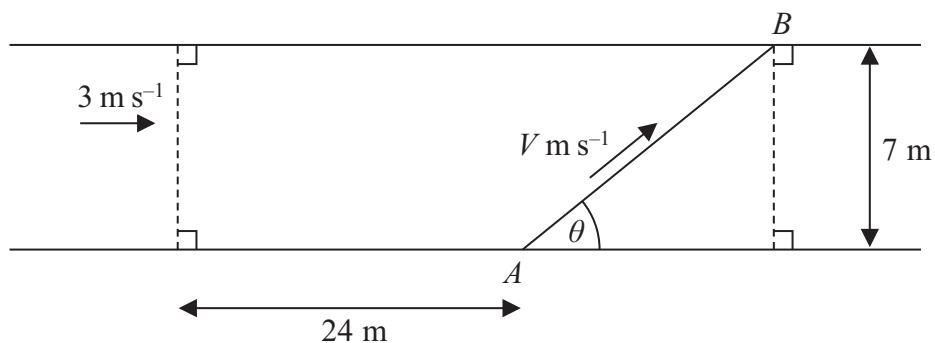


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.

John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

- (b) find the minimum value of V . (2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB . (3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$. (6)



